MATH 2E Prep: Determinants

1. Calculate $\begin{vmatrix} 8 & 5 \\ 0 & 12 \end{vmatrix}$.

Solution:

$$\begin{vmatrix} 8 & 5 \\ 0 & 12 \end{vmatrix} = 8 \times 12 - 5 \times 0 = 96$$

2. Calculate $\begin{vmatrix} 2 & 3 & 0 \\ 0 & 1 & 5 \\ -2 & 1 & 4 \end{vmatrix}$.

Solution:

$$\begin{aligned} \begin{vmatrix} 2 & 3 & 0 \\ 0 & 1 & 5 \\ -2 & 1 & 4 \end{vmatrix} &= 2\begin{vmatrix} 1 & 5 \\ 1 & 4 \end{vmatrix} - 3\begin{vmatrix} 0 & 5 \\ -2 & 4 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 \\ -2 & 1 \end{vmatrix} \\ &= 2 \times (4-5) - 3 \times (0+10) + 0 \times \begin{vmatrix} 0 & 1 \\ -2 & 1 \end{vmatrix} \\ &= -2 - 30 + 0 \\ &= -32 \end{aligned}$$

3. Calculate $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ y & z & x \end{vmatrix}$.

Solution:

$$\begin{aligned} \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ y & z & x \end{vmatrix} &= \begin{vmatrix} y & z \\ z & x \end{vmatrix} - \begin{vmatrix} x & z \\ y & x \end{vmatrix} + \begin{vmatrix} x & y \\ y & z \end{vmatrix} \\ &= xy - z^2 - (x^2 - yz) + xz - y^2 \\ &= xy + yz + xz - x^2 - y^2 - z^2 \end{aligned}$$